

Dear Lou,

Here is a quick history of integrated math programs that may help give you useful background.

In many of the highest achieving countries in mathematics, it was noticed that the traditional sequence - Algebra I, Geometry, Algebra II, Trigonometry, Pre-calculus, Calculus - was not present. Rather, there were courses that we might call Integrated Math I, Integrated Math II, Integrated Math III, and then Pre-calculus and Calculus. These Integrated courses follow a different ordering, though they cover the same topics to at least the same depth as the traditional programs, Algebra I, Geometry, Algebra II, Trigonometry, that they replace. They might start with the coordinate line and coordinate plane, and basic geometric constructions in the plane. Then they might do the basic algebra of linear equations. Then they might return to relate the graphs of linear equations with lines in the plane, and justify this connection by studying the classic properties of similar triangles in the plane. And so on.

The point is that in the "integrated" courses in the high achieving countries, instead of just doing all the algebra necessary to understand key aspects of geometry, and constantly having to say "this material is very important, but you will have to take my word for now," they try to bring in the relevant concepts and material so that geometry and algebra support each other directly.

This is a process that is mathematically sensible and clearly tends to have good outcomes. However, about 20 years back, when we started to design similar courses for our students in the United States, a new pressure had been added - it was required to make mathematics accessible to ALL students, rather than just elite students. So the integrated courses that the National Science Foundation, (NSF), funded had the objective of making the material in the high school mathematics sequence accessible to all students - a highly laudable goal, and one that is achieved in the high-achieving countries.

Unfortunately, the research that underlies the success of the integrated programs in the high achieving countries was not considered relevant by the authors that NSF supported in this country, and the resulting programs were written - in effect - by seat of the pants methods.

When we research mathematicians looked at the resulting programs we were dismayed. Instead of developing a core piece of algebra or geometry fully and then applying it, as is done in the programs in high achieving countries, there would be one lesson on one topic, then a lesson on a completely different topic, and no connections were ever made. To our eyes, these programs looked like confusing messes. Moreover, they did not cover the entire curriculum in algebra, geometry, and beyond, which had been covered in the traditional sequence in contradistinction to the foreign programs. In fact, they tended to only cover small parts of it.

Much was made of "problem solving." It was constantly pointed out that these NSF funded programs developed "problem solving" skills in students, but this seemed very strange to research mathematicians when we looked at problem solving pages like this one from the second course in IMP (Interactive Math Program), one of the NSF funded curricula.

# Homework 1

# What to Put It In?

All sorts of containers are used for everyday goods. Containers are made out of different materials, have different sizes and shapes, and are used for different purposes. Look around your home for different kinds of containers. If possible, stop at a supermarket or other store for more ideas.

1. What shapes did you find? Make sketches of the different containers you saw.
2. What kinds of materials were used?
3. What units of measurement were used to indicate how much a container holds?
4. What criteria do you think manufacturers use to decide what kind of container to use for their products?
5. Choose a specific container and write about *one* of these two topics:
  - How the container could have been designed better
  - Why the manufacturer designed the container that way



I think it is evident that whatever the authors of this program might mean by problem solving, the above example shows that it has virtually nothing to do with problem solving in mathematics or the sciences.

What research mathematicians really do is to solve problems. For example, I currently work on problems in robotics and in protein folding. The one thing that I know is that one cannot solve problems without understanding as much as possible about the area where the problem occurs and understanding related areas as well. The very first requirement is to understand what the problem is asking and the context in which the problem arises. These NSF funded programs seemed to consistently trivialize both mathematical content and contexts, so we doubted that they would be successful.

In particular, we asked for evidence that the programs actually improved student outcomes in mathematics. In California in the 1990's where these NSF funded programs were extensively and increasingly in use, we noted that the remediation rates in mathematics in the California State University system (that enrolls the top 30% of high school graduates in the state) skyrocketed from 23% to 54% during the period 1989 - 1997. There were some changes in the criteria for high school graduation during this period, so the change in remediation rates is not correlated just with these integrated programs. However, California moved away from the NSF funded integrated programs starting in 1998, and the current remediation rate in mathematics is about 36%. So we were dubious that there would be any evidence that could be provided to us that would show the success of these programs. Indeed, when the NSF funded programs gave us their research and we looked at it closely, it fell apart. Not one methodologically valid paper actually showed an educationally significant improvement in outcomes for students in these classes. About the best that could be said in any of the valid studies was that in a few of them - for the measures used - there did not seem to be worse outcomes than for traditional programs. However, the catch is "the measures used." They were not measures like the ACT exams or the SAT exams that correlate strongly with success in college, but tended to be self-constructed and non-validated assessments written by the program developers, or exams like the New Standards Reference Exam where there has been no work on correlating test results with long term student outcomes.

It was exactly this failure that led to the development of the What-Works Clearinghouse at the Institute for Education Science in the U.S. Department of Education. I am a member of the Presidential Board that oversees the Institute for Education Science, and we are currently very concerned with the What-Works Clearinghouse, since they are having problems obtaining reliable evaluations. As it is, virtually all of the papers they have analyzed have been judged unreliable, so they have reported on papers that one or two evaluators viewed as marginally sound. However, when a number of us on the Board actually looked at even these papers, we found that they were very questionable. Part of the problem is that there are simply too few people in the area that understand what solid methodology in this area looks like.

Moreover there is a growing body of methodologically sound research - for example the paper by R. Hill and T. Parker that just appeared in the journal *The American Mathematical Monthly* published by the Mathematics Association of America - that shows that implementation of these programs is associated with poorer college performance in mathematics. The Hill-Parker paper studies all the students from the high schools in Michigan for a number of years who attended Michigan State University, and compares the outcomes for the students from CorePlus high schools with those from other high schools. The paper shows

that there was no difference in mathematics outcomes at Michigan State for the two groups before the introduction of CorePlus, but after that there was a steady and highly significant decline in student outcomes for the CorePlus students, while the outcomes for the rest of the students remained essentially unchanged.

The Math Monthly of the MAA has very high standards and very few papers in math education are accepted for it. The refereeing is extensive, so when a paper in this area does appear in The Math Monthly, it needs to be taken very seriously.

In short, though there is no doubt that properly constructed integrated programs are preferable to the traditional sequence in mathematics, there is equally no doubt that the current programs of this kind available in the United States are experimental, and there is no reliable research that shows they are effective. At the same time there is a growing body of research that casts serious doubts on the effectiveness of any of the NSF funded curricula, including those such as the TERC program, Investigations in Data, Number and Space, and the middle school program CMP (Connected Math Project), though I have not discussed the issues with these programs above.

I will try to put together some comments on the critical importance of a solid math background for students these days in a following letter. But there is overwhelming evidence that a solid background in mathematics is, besides the ability to read well, the single most essential thing that students must obtain in their K-12 educations currently. Without it, as the unfortunate situation with your son shows, high school graduates have only limited options, and most of them are constrained to low paid futures. I think that, these days, it is legitimate to say "math is money," and failure to provide a solid math education in K-12 is nothing short of criminal.

Yours,  
Jim Milgram